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390. Proposed by E. B. ESCOTT, University of Michigan.

Sum the series

$$\frac{1}{1}+\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{5}{16}+\frac{8}{32}+\cdots,$$

where each numerator is the sum of the two preceding and the denominators are in geometrical progression.

SOLUTION BY R. D. CARMICHAEL, Indiana University.

The series may be written in the form

$$S = 1 + \sum_{n=1}^{\infty} \frac{u_n}{2^n}, \tag{1}$$

where

$$u_{n+2} - u_{n+1} - u_n = 0, \qquad u_1 = 1, \qquad u_2 = 2. \tag{2}$$

We first solve the recurrence relation (2) without restriction of initial values. Substituting ρ^n for u_n, we find that (2) will be satisfied if

$$\rho^2 - \rho - 1 = 0;$$

that is, if ρ has either of the values

$$\rho_1 = \frac{1 + \sqrt{5}}{2}, \qquad \rho_2 = \frac{1 - \sqrt{5}}{2}.$$

The general solution of (2), without restriction of initial values, is therefore

$$u_n = \alpha\rho_1{}^n + \beta\rho_2{}^n,$$

where α and β are constants.

We must now determine α and β so that $u_1 = 1$ and $u_2 = 2$; that is, so that

$$\alpha\rho_1 + \beta\rho_2 = 1,$$

$$\alpha\rho_1{}^2 + \beta\rho_2{}^2 = 2.$$

Thus we have readily

$$\alpha = \tfrac{1}{10}(5 + \sqrt{5}), \qquad \beta = \tfrac{1}{10}(5 - \sqrt{5}).$$

Putting in (1) the value of u_n thus determined, we have

$$S = 1 + \tfrac{1}{10}(5 + \sqrt{5})S_1 + \tfrac{1}{10}(5 - \sqrt{5})S_2, \tag{3}$$

where

$$S_1 = \sum_{n=1}^{\infty} \frac{(1 + \sqrt{5})^n}{2^n \cdot 2^n}, \qquad S_2 = \sum_{n=1}^{\infty} \frac{(1 - \sqrt{5})^n}{2^n \cdot 2^n}.$$

Summing these geometric series, we have

$$S_1 = 2 + \sqrt{5}, \qquad S_2 = 2 - \sqrt{5}.$$

Substituting in (3), we obtain

$$S = 4,$$

the result sought.

Also solved by H. A. Levy, A. M. Harding, J. A. Colson, Levi S. Shively, H. C. Feemster, Elmer Schuyler, and the Proposer.

391. Proposed by C. N. SCHMALL, New York, N. Y.

Show that the roots of the quadratic

$$ax^2 + 2bx + c = 0$$

are imaginary if a, b, c are in harmonic progression and have the same sign.

Solution by S. W. Reaves, University of Oklahoma.

Since a, b, c are in harmonic progression,

$$b = \frac{2ac}{a + c}.$$

The discriminant of the given quadratic then becomes

$$b^2 - ac = \left(\frac{2ac}{a + c}\right)^2 - ac = -\,ac\left(\frac{a - c}{a + c}\right)^2,$$

which is negative since a and c have the same sign. The discriminant being negative, the roots are imaginary.

Also solved by Walter C. Eells, Elmer Schuyler, F. M. Morgan, H. C. Feemster, J. A. Colson, and A. M. Harding.

Additional solutions of 387 were received from R. M. Mathews, Horace Olson, and G. Y. Sosnow, after the December issue had gone to press.

Note. We have solutions of all problems proposed during 1913 in this section up to the September number, except 385, which was published in February.

GEOMETRY.

419. Proposed by S. LEFSCHETZ, University of Nebraska.

Given a circle and a tangent to it. To find a point on its circumference such that the sum of its distances to the tangent and its point of contact shall be equal to a given length.

Solution by S. W. Reaves, University of Oklahoma.

Let ORN be the given circle, OT the given tangent, and O the point of contact. Denote the given length by l.